Coimisiún na Scrúduithe Stáit

State Examinations Commission

Leaving Certificate 2022

Marking Scheme

Applied Mathematics

Ordinary Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

General Guidelines

1. Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	(-1)
Blunders	- mathematical errors	(-3)
Misreading	- if not serious	(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: (A2)

2. The marking scheme shows one correct solution to each question.
In many cases there are other equally valid methods.

3. Examiners are expected to annotate parts of the responses as directed at the marking conference. (See below.)

Symbol	**Name**	**Use**
✗	Cross	Incorrect element
✓	Tick	Correct element (0 marks)
A2	Attempt-2	2 marks
3	3	3 marks
4	4	4 marks
✓ 5	Tick-5	Correct element (5 marks)
✓ 10	Tick-10	Correct element (10 marks)
	Horizontal wavy line	To be noticed
	Vertical wavy line	Additional page

4. Bonus marks at the rate of 5% of the marks obtained will be given to a candidate who answers entirely through Irish and who obtains 75% or less of the total mark available (i.e. 187 marks or less). In calculating the bonus to be applied decimals are always rounded down, not up ¬ e.g., 4.5 becomes 4; 4.9 becomes 4, etc. See below for when a candidate is awarded more than 187 marks.

Marcanna Breise as ucht freagairt trí Ghaeilge

Léiríonn an tábla thíos an méid marcanna breise ba chóir a bhronnadh ar iarrthóirí a ghnóthaíonn níos mó ná 75% d'iomlán na marcanna.

N.B. Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don scrúdú. Ba chóir freisin an marc bónais sin **a shlánú síos**.

Tábla 250 @ 5%
Bain úsáid as an tábla seo i gcás na n-ábhar a bhfuil 250 marc san iomlán ag gabháil leo agus inarb é 5% gnáthráta an bhónais.

Bain úsáid as an ngnáthráta i gcás 187 marc agus faoina bhun sin. Os cionn an mharc sin, féach an tábla thíos.

Bunmharc	Marc Bónais
188 - 190	9
191 - 196	8
197 - 203	7
204 - 210	6
211 - 216	5

Bunmharc	Marc Bónais
217 - 223	4
224 - 230	3
231 - 236	2
237 - 243	1
244 - 250	0

1. A car starts from rest at point P and accelerates uniformly to a speed of 20 m s^{-1} in a time of 5 s. The car then decelerates to 15 m s^{-1} with a constant deceleration of 0.5 m s^{-2}. It continues at this speed for a further 8 s. The car then decelerates with a *different* constant deceleration until it comes to rest at point Q.

The total time for the journey from P to Q was 35 s.

(i) Sketch a speed-time graph of the motion of the car as it travels from P to Q.

(ii) Calculate the acceleration of the car during the first 5 s.

(iii) Calculate the distance travelled by the car during the first 5 s.

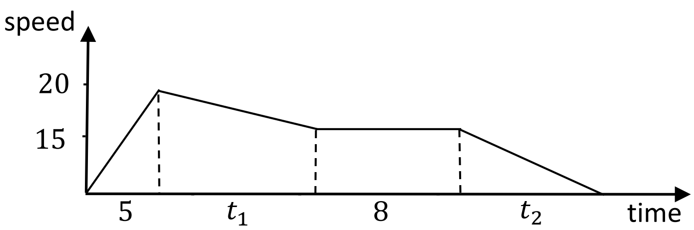
(iv) Calculate the distance travelled by the car while decelerating from 20 m s^{-1} to 15 m s^{-1}.

(v) Calculate the total distance between P and Q.

A motorcycle passes point P with a speed of u m s^{-1} at the same instant that the car starts from P. The motorcycle travels at speed u for 20 s before decelerating to rest so that it finishes at point Q at the same instant as the car.

(vi) Find the value of u.

(i)



(10)

(ii) $a = \frac{20}{5} = 4 \text{ m s}^{-2}$ (10)

(iii) $s_1 = \frac{1}{2} \times 5 \times 20 \ = 50 \text{ m.}$ (5)

(iv)

$$v^2 = u^2 + 2as$$
$$15^2 = 20^2 + 2\left(-\frac{1}{2}\right)s_2$$
$$s_2 = 175 \text{ m.}$$

(5)

(v)

$$\frac{5}{t_1} = \frac{1}{2} \ \Rightarrow t_1 = 10$$

$$t_2 = 35 - 5 - 10 - 8 = 12$$

$$s = 50 + 175 + 8(15) + \frac{1}{2} \times 12 \times 15 = 435 \text{ m.}$$

(10)

(vi)

$$20u + \frac{1}{2}(15)(u) = 435$$

$$u = 15.8 \text{ m s}^{-1}$$

(10) (50)

2. At noon, ship A is 100 km west of ship B. Ship A has a speed of $3\sqrt{3}$ km h^{-1} in a direction east 30° south. Ship B has a speed of $4\sqrt{2}$ km h^{-1} in a direction west 45° south.

(i) Draw a diagram to show the positions and velocities of the two ships.

(ii) Write the velocity of ship A and the velocity of ship B in terms of $\vec{i}$ and $\vec{j}$.

(iii) Calculate $\overrightarrow{v_{AB}}$, the velocity of ship A relative to ship B, in terms of $\vec{i}$ and $\vec{j}$.

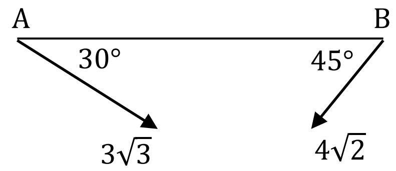
(iv) Calculate the magnitude of $\overrightarrow{v_{AB}}$ to one decimal place.

(v) Calculate the direction of $\overrightarrow{v_{AB}}$ to one decimal place.

(vi) Show that the shortest distance between the ships is 16.3 km.

(vii) Calculate the time when the ships are closest to each other.

(i)



(10)

(ii) $\vec{V_A} = 4.5\,\vec{i} - \frac{3}{2}\sqrt{3}\,\vec{j}$ (5)

$\vec{V_B} = -4\,\vec{i} - 4\,\vec{j}$ (5)

(iii) $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

$\vec{V}_{AB} = 8.5\,\vec{i} + 1.4\,\vec{j}$ (5)

(iv) $\left|\vec{V}_{AB}\right| = \sqrt{8.5^2 + 1.4^2} = 8.6$ (5)

(v) $\tan\alpha = \frac{1.4}{8.5}$

$\alpha = \text{east } 9.4° \text{ north}$ (5)

(vi) $d = 100 \times \sin 9.4 = 16.3 \text{ km}$ (5)

(vii) $t = \frac{100 \times \cos 9.6}{8.6} = 11.47 \text{ h}$ (10) (50)

3. **(a)** A particle is projected from a point A on a horizontal plane with an initial velocity of $25\vec{\imath} + 15\vec{\jmath}$ m s^{-1}. The particle lands at a point B, which is a distance d m from A.

Calculate

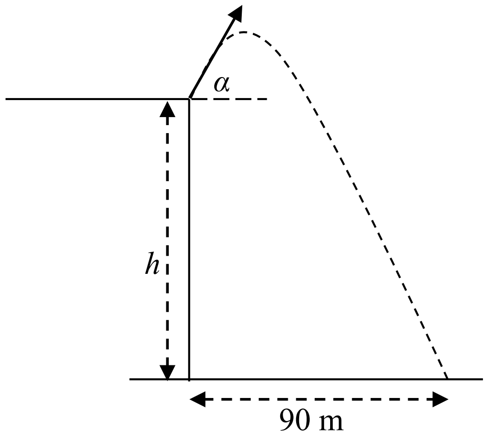
(i) the time of flight of the particle

(ii) the value of d

(iii) the maximum height of the particle above the horizontal plane

(iv) the speed of the particle after 1.75 s.

(b) A particle is projected from the edge of a vertical cliff of height h m. The particle is projected with a speed of 26 m s^{-1} at an angle α above the horizontal, as shown in the diagram. Tan $\alpha = \frac{5}{12}$. The particle lands 90 m from the base of the cliff.



Calculate

(i) the time taken for the particle to land

(ii) the value of h.

(a) **(i)** $\vec{r_j} = 0 \;\Rightarrow\; 15t - 5t^2 = 0$

$t = 3$ s (10)

(ii) $d = 25 \times 3 = 75$ m (5)

(iii) $t = 1.5$

$h = 15(1.5) - 5(1.5)^2$

$h = 11.25$ m (10)

(iv) $v_x = 25$

$v_y = 15 - 10(1.75) = -2.5$

$v = \sqrt{25^2 + (-2.5)^2} = 25.12$ m s^{-1}. (10)

(b) **(i)** $90 = 26\cos\alpha \times t$

$t = \frac{90}{26\cos\alpha} = \frac{90}{24} = 3.75$ s. (10)

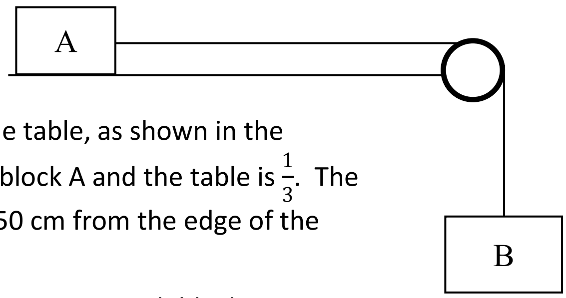
(ii) $26\sin\alpha \times t - 5t^2 = -h$

$h = 5t^2 - 10t$

$h = 5(3.75)^2 - 10(3.75)$

$h = 32.8$ m. (5) (50)

4. **(a)** A block A, of mass 3 kg, is at rest on a rough horizontal table. It is connected to block B, of mass 7 kg, by a light inelastic string that passes over a smooth pulley at the edge of the table, as shown in the diagram. The coefficient of friction between block A and the table is $\frac{1}{3}$. The system is released from rest when block A is 50 cm from the edge of the table.



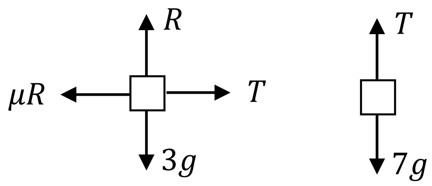
(i) Show on separate diagrams the forces acting on each block.

(ii) Show that the common acceleration of the blocks is 6 m s^{-2}.

(iii) Calculate the tension in the string.

(iv) Calculate the speed of block A when it reaches the edge of the table.

(i)



(10)

(ii)

$$7g - T = 7a$$

$$T - g = 3a$$

$$a = 6 \ \text{m s}^{-2}$$ (15)

(iii)

$$T = 3a + g$$

$$T = 28 \ \text{N}$$ (5)

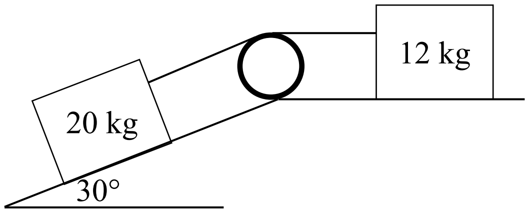
(iv)

$$v^2 = u^2 + 2as$$

$$v^2 = 0 + 2 \times 6 \times 0.5$$

$$v = \sqrt{6} = 2.45 \ \text{m s}^{-1}$$ (5) (35)

(b) Blocks of mass 20 kg and 12 kg are connected by a light inelastic string which passes over a smooth pulley, as shown in the diagram. The 20 kg block lies on a smooth surface which is inclined at 30° to the horizontal. The 12 kg block lies on a smooth horizontal surface.



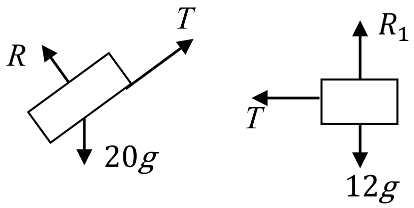
The system is released from rest.

(i) Calculate the acceleration of the system.

The surface beneath the 12 kg mass is replaced with another surface which has a coefficient of friction of μ.

(ii) Calculate the minimum value of μ so that the system does not move.

(i)



$$200\sin 30 - T = 20a$$

$$T = 12a$$

$$a = \frac{25}{8} = 3.125 \text{ m s}^{-2}. \qquad (10)$$

(ii)

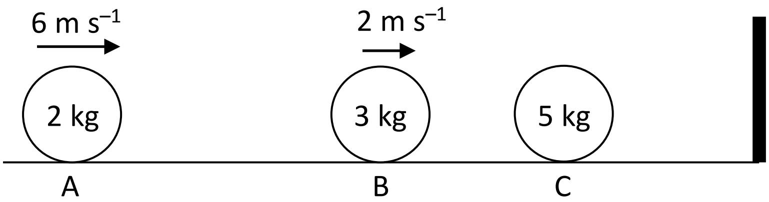
$$200\sin 30 - T = 20a$$

$$T - \mu(12g) = 12a$$

$$100 - \mu(120) = 32a$$

$$a = 0 \quad \Rightarrow \quad \mu = \frac{5}{6}. \qquad (5) \qquad (15)$$

5. Three smooth spheres are positioned on a smooth horizontal table as shown. Sphere A, of mass 2 kg, moves towards sphere B with a velocity of 6 m s^{-1}. Sphere B, of mass 3 kg, has a velocity of 2 m s^{-1} in the same direction as sphere A. Sphere C, of mass 5 kg, is at rest. Sphere C is 12 m away from a wall.



The coefficient of restitution for the collision between sphere A and sphere B is $\frac{1}{4}$.

Calculate

(i) the speed of sphere A and the speed of sphere B after they collide

(ii) the loss of kinetic energy due to this collision

(iii) the magnitude of the impulse imparted to sphere A as a result of this collision.

Sphere B then collides with sphere C. Sphere B comes to rest as a result of this collision.

(iv) Calculate the value of the coefficient of restitution, *e,* between spheres B and C.

(v) Calculate the time C takes to reach the wall.

(i) PCM $2(6) + 3(2) = 2v_1 + 3v_2$ (5)

$2v_1 + 3v_2 = 18$

NEL $v_1 - v_2 = -\frac{1}{4}(6 - 2)$ (5)

$3v_1 - 3v_2 = -3$

$v_1 = 3 \qquad v_2 = 4$ (10)

(ii) $\text{KE}_\text{B} = \frac{1}{2}(2)(6)^2 + \frac{1}{2}(3)(2)^2 = 42$

$\text{KE}_\text{A} = \frac{1}{2}(2)(3)^2 + \frac{1}{2}(3)(4)^2 = 33$

$\text{KE}_\text{B} - \text{KE}_\text{A} = 42 - 33 = 9\ \text{J}$ (10)

(iii) $I = |2 \times 3 - 2 \times 6| = 6\ \text{N s}$ (5)

(iv) PCM $3(4) + 5(0) = 0 + 5v_3$ (5)

$v_3 = 2.4$

NEL $0 - 2.4 = -e(4 - 0)$ (5)

$e = 0.6$

(v) $t = \frac{12}{2.4} = 5\ \text{s}.$ (5) (50)

6. **(a)** Particles of weights 5 N, 2 N, 3 N and 10 N are placed at the points (p, –1), (–6, q), (4, –5) and (5, p) respectively.

The co-ordinates of the centre of gravity of the system are (3, q).

Calculate

(i) the value of p

(ii) the value of q.

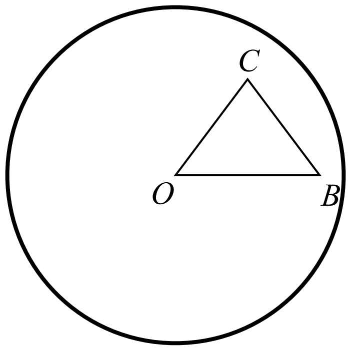
(i) $$3 = \frac{5p+2(-6)+3(4)+10(5)}{20}$$

$p = 2$ (10)

(ii) $$q = \frac{5(-1)+2q+3(-5)+10p}{20}$$

$q = 0$ (10) (20)

(b) A uniform circular lamina with centre O and radius 7 cm has isosceles triangle OBC removed from it, where $|OB|$ = 6 cm and $|OC|$ = $|BC|$= 5 cm. Centre O and point B have co-ordinates
(0, 0) and (6, 0) respectively.



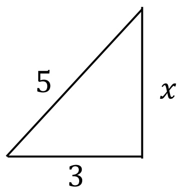
The remaining portion of the lamina has its centre of gravity at point P.

(i) Show that point C has co-ordinates (3, 4).

(ii) Calculate the co-ordinates of P to two decimal places.

(iii) Calculate $|OP|$ to two decimal places.

(i)



$$x^2 = \sqrt{5^2 - 3^2} = 4$$

C (3, 4) (10)

(ii)

	area	c.g.	
circle	49π	$(0,\ 0)$	
OBC	12	$\left(3,\ \frac{4}{3}\right)$	(5)
lamina	$49\pi - 12$	$(\bar{x},\ \bar{y})$	(5)

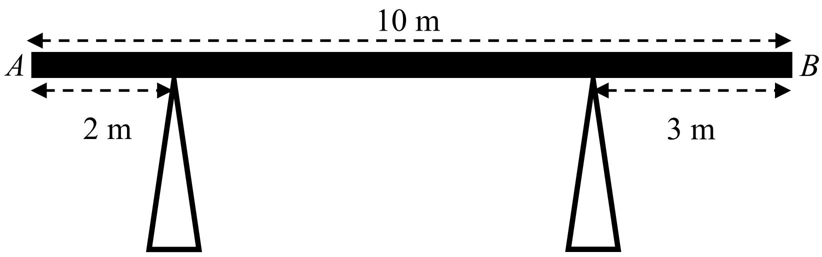
$$(49\pi - 12)\bar{x} = 49\pi \times 0 - 12 \times 3$$
$$\bar{x} = -0.25$$

$$(49\pi - 12)\bar{y} = 49\pi \times 0 - 12 \times \frac{4}{3}$$
$$\bar{y} = -0.11$$

$$(-0.25, -0.11)$$ (5)

(iii) $|OP| = \sqrt{0.25^2 + 0.11^2} = 0.27\ \text{cm}$ (5) (30)

7. **(a)** A rod *AB*, of length 10 m and weight 800 N rests on two vertical supports, as shown in the diagram. The first support is placed 2 m from *A* and the second support is placed 3 m from *B*.

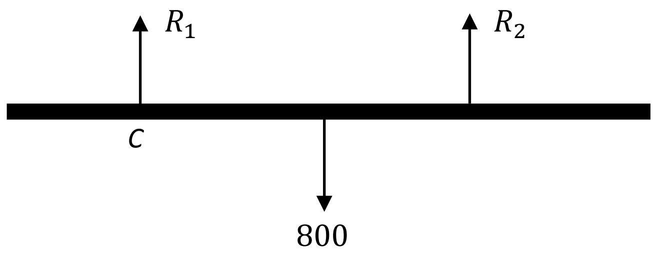


The rod is at rest and is in horizontal equilibrium.

(i) Draw a force diagram showing all the forces acting on the rod.

(ii) Calculate the normal reactions between the rod and each of the vertical supports.

(i)



(10)

(ii) C: $R_2 \times 5 = 800 \times 3$

$$R_2 = 480 \text{ N}$$ (10)

$$R_1 + R_2 = 800$$
$$R_1 + 480 = 800$$
$$R_1 = 320 \text{ N}$$ (5)

(25)

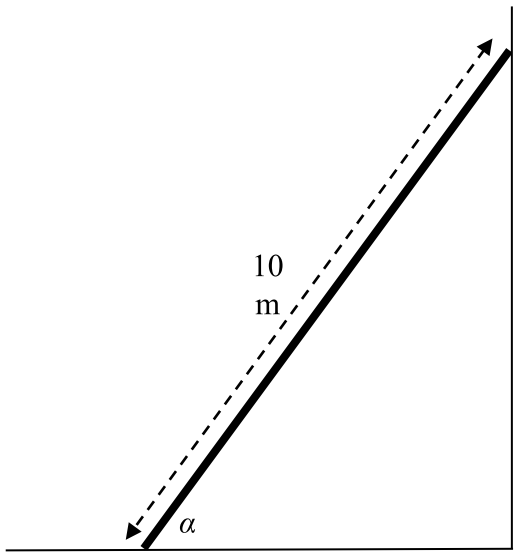
(b) A uniform ladder of length 10 m and weight 120 N rests on rough horizontal ground and leans against a smooth vertical wall.

The ladder is on the point of slipping when it makes an angle α with the ground, where $\tan \alpha = \frac{4}{3}$.

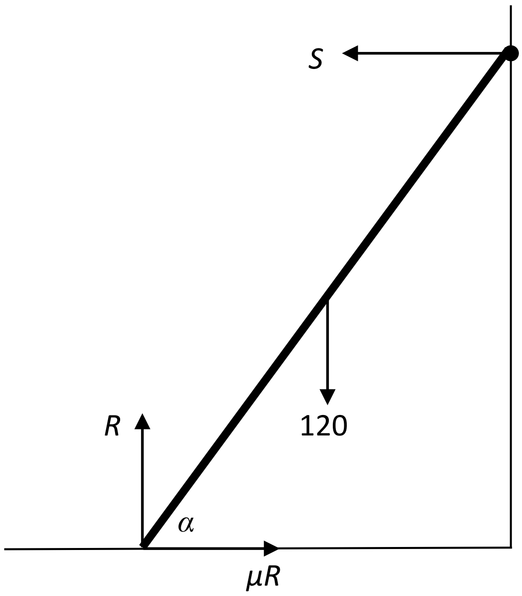
(i) Draw a force diagram showing all the forces acting on the ladder.

(ii) Calculate the normal reaction between the ladder and the wall.

(iii) Find the coefficient of friction between the ladder and the ground.



(i)



(5)

(ii) $S \times 8 = 120 \times 3$ (10)

$S = 45$ N (5)

(iii) $R = 120$

$\mu R = S$
$\mu \times 120 = 45$

$\mu = \frac{3}{8}$ (5) (25)

8. **(a)** A particle of weight 40 N moves with uniform circular motion in a horizontal circle of radius 2 m. It completes 50 rotations in one minute.

Calculate

(i) the mass of the particle

(ii) how many seconds the particle takes to complete one rotation

(iii) the angular velocity of the particle

(iv) the linear velocity of the particle

(v) the centripetal force acting on the particle.

(i) $m = \frac{40}{10} = 4 \text{ kg}$ (5)

(ii) $T = \frac{60}{50} = 1.2 \text{ s}$ (5)

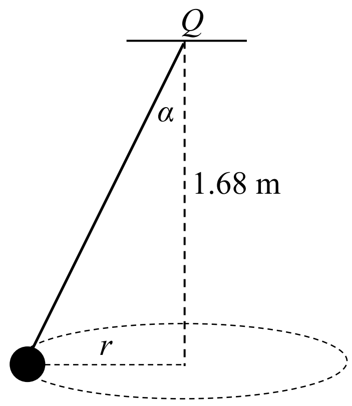
(iii) $T = \frac{2\pi}{\omega} = 1.2 \text{ s}$

$\omega = \frac{2\pi}{1.2} = \frac{5\pi}{3} \text{ rad s}^{-1}$ (5)

(iv) $v = r\omega = 2 \times \frac{5\pi}{3} = 10.5 \text{ m s}^{-1}$ (5)

(v) $F = mr\omega^2 = 4 \times 2 \times \left(\frac{5\pi}{3}\right)^2 = 219.3 \text{ N}$ (5) (25)

(b) A particle of mass 3 kg is connected to a fixed point *Q* by a light inelastic string. The particle describes a horizontal circle of radius *r* m. The centre of the circle is 1.68 m vertically below *Q*, as shown in the diagram. The string makes an angle α with the vertical, where $\tan\alpha = \frac{7}{24}$.



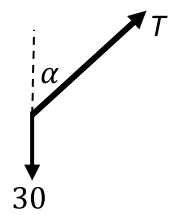
(i) Draw a force diagram showing all the forces acting on the particle.

(ii) Calculate *r*.

(iii) Calculate the tension in the string.

(iv) Calculate the speed of the particle.

(i)



(5)

(ii) $\tan\alpha = \frac{7}{24} = \frac{r}{1.68}$

$r = 0.49 \text{ m}$ (10)

(iii) $T\cos\alpha = 30$

$T \times \frac{24}{25} = 30$

$T = 31.25 \text{ N}$ (5)

(iv) $T\sin\alpha = \frac{mv^2}{r}$

$31.25 \times \frac{7}{25} = \frac{3v^2}{0.49}$

$v = 1.2 \text{ m s}^{-1}$ (5) (25)

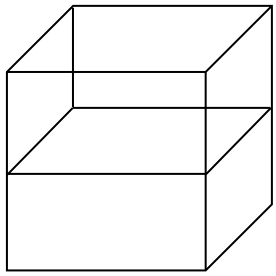
9. **(a)** A cubic tank of side 50 cm and mass 8 kg is designed to hold a liquid of density 1200 kg m^{-3}.

The tank is *half-filled* with this liquid.

Calculate

(i) the total weight of the tank plus liquid

(ii) the pressure the liquid exerts on the bottom of the tank.



Another three litres of the liquid are added to the tank.

(iii) Calculate the total pressure the liquid now exerts on the bottom of the tank.

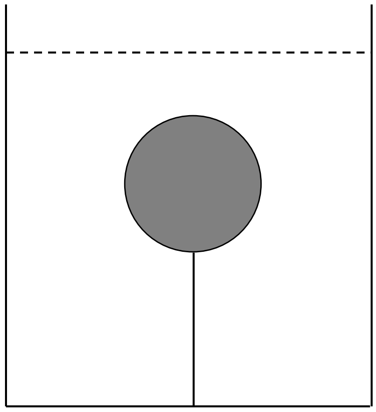
Note*: 1 litre = 0.001 m^3*

(b) A solid sphere of radius *r* cm and density 300 kg m^{-3} is completely immersed in a tank containing a liquid of density 1500 kg m^{-3}.

The sphere is attached to the bottom of the tank by a string which has a tension of 11 N.

(i) Draw a force diagram showing all the forces acting on the sphere.

(ii) Calculate *r*.



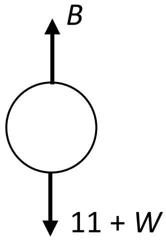
(a) **(i)** $$W = 80 + 1200\left\{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{4}\right\} \times 10 = 830 \text{ N}$$ (10)

(ii) $$P = \rho g h = 1200 \times 10 \times \frac{1}{4} = 3000 \text{ Pa}$$ (5)

(iii) $$\frac{1}{2} \times \frac{1}{2} \times h = 0.003 \quad \Rightarrow \quad h = 0.012$$

$$P = 1200 \times 10 \times 0.262 = 3144 \text{ Pa}$$ (10) (25)

(b) **(i)**



(5)

(ii) $$B = 1500 \times \left(\frac{4}{3}\pi r^3\right) \times 10 = 20000\pi r^3$$ (5)

$$W = 300 \times \left(\frac{4}{3}\pi r^3\right) \times 10 = 4000\pi r^3$$ (5)

$$B = 11 + W$$ (5)

$$20000\pi r^3 = 11 + 4000\pi r^3$$

$$r = 0.06 \text{ m} = 6 \text{ cm.}$$ (5) (25)